STATISTICAL ANALYSIS OF THE INFLUENCE OF THE CHEMICAL COMPOSITION AND HARDNESS ON THE DEFORMATIONS OF A NORMALLY LOADED POINT CONTACT

Alexandru Potorac, Dorel Prodan, Luminiţa Irimescu, Florina Ciornei
"Stefan cel Mare" University of Suceava

Keywords: hardness, chemical composition, elastic deformations, plastic deformations, point contact

Abstract: The theory of the probabilities and the mathematical statistics is a modern and useful tool in developing models to characterize technical phenomena. Through the specialized soft "STATISTICA", the paper is approaching an analysis of the influence of the hardness and chemical structure of the samples upon the elastic as well as plastic contact deformations.

1. General Considerations

The Theory of Elasticity is based on the fundamental assumption of the proportionality between the deformation and the load. In the same time, for a cycle, the loading and unloading takes place after the same straight-line [1].

For hertzian contacts, the phenomenon is more complex due to a semi-cubical dependency between the deformation and the load. In addition, the loading and unloading aren't produced after a same curve.

The analysis presented above is referred to the problem of the normal point contact. For this matter is considered the contact model from Figure 1.

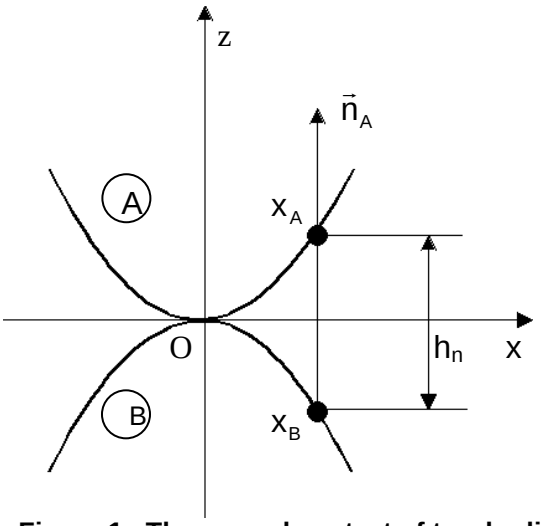


Figure 1: The normal contact of two bodies.

In undistorted state, two points X_A and X_B belonging to the surfaces of the bodies A and B are considered to be in opposition when they belong to the same normal $\vec{n}_A$ to the initial tangent plan. The distance between these two points of the bodies in undistorted state is h_n being measured on the normal $\vec{n}_A$.

In the case of the real point contacts, besides the elastic deformation (δ_{el}) of the bodies in contact it also appears a plastic component (δ_{pl}). Measuring the total deformation (δ_{tot}) and the plastic deformation we can calculate the elastic deformation:

$$\delta_{el} = \delta_{tot} - \delta_{pl} \qquad (1)$$

2. Experimental results

Using the test-rig from Figure 2 [2], the values of the elastic and plastic deformation at the same load for 32 different test samples were determined. These values are obtained according to the mechanical features (hardness) and chemical composition (C, Mn, Cr) of the samples. The experimental results were obtained in static conditions, the load being applied ascending slow.

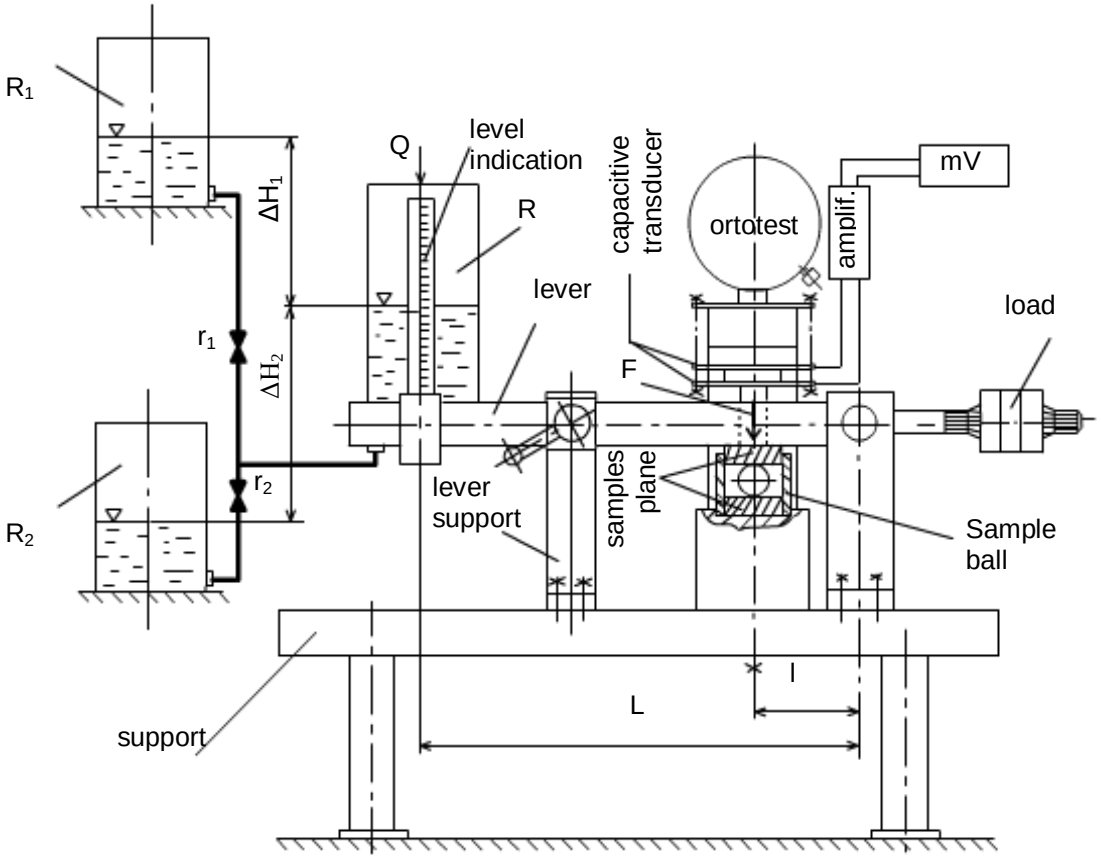


Figure 2. Test rig for loading the contact with normal load (principle scheme).

The main component parts of the stand are:
-the base;
-the system of application, amplification and measurement of the load;
-the system of amplification and measurement of the deformation;
-the system of fixing the samples (test bars).

The results obtained show a relative rapid stabilization of the deformations for the high rank cycles, Figure 3. We can also notice that the deformation remains prevailingly elastic, excepting the first loading cycles.

That's why there will be hereinafter analyzed only the results of the first charging-discharging cycle.

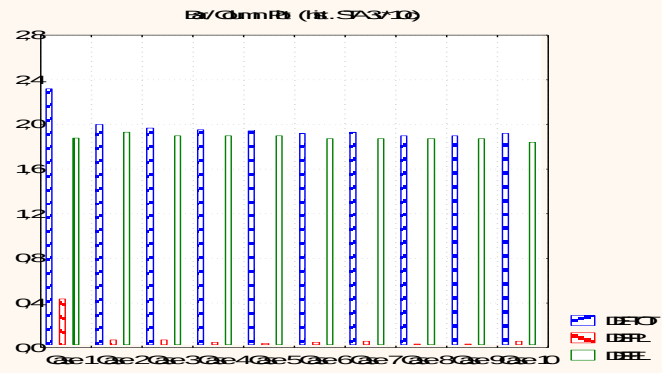


Figure 3: The histograms of the deformations according to the loading cycles.

The variation of the elastic and plastic deformation according to the test number, for the first loading cycle is presented in the Figure 4.

From this figure we can notice that the variation of the elastic and plastic deformation has the same form, the first one being 3 ÷5 times bigger.

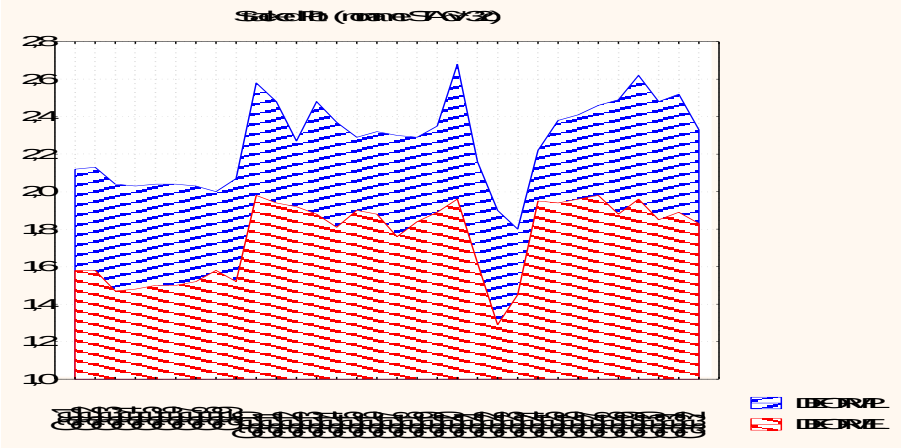


Figure 4. The variation of the deformations according to the test type.

3. Establishing of the random variables, calculus of the statistic parameters and verification of the distributions' normality.

There are determined the values of the variables: elastic deformation, plastic deformation, hardness, and also the content of Carbon, Manganese and Chrome for 32 samples. We consider the deformations as dependent variables and the hardness and the percentage of the main alloying elements (C, Mn and Cr) as independent variables.

Among the analyzed statistic parameters we can mention: the average, the median, the minimum value, the maximum value, the inferior quartile, the superior quartile, the variant, the standard deviation, the standard error, Skewness, the Skewness standard error, Kurtosis, the Kurtosis standard error, [3].

We consider that the elastic deformation, the plastic deformation and the hardness are random variables with a normal distribution. The main alloying elements of the steel of tested ball-bearing don't present a sufficient number of distinctive values in order to appreciate that they have or not a normal distribution.

The verification, for illustration, of the normality of the plastic deformation variable distribution is done with the Shapiro-Wilk test [4]. The result of the Shapiro-Wilk test

(W=0.97094, p<0.5257) for the DEFORMPL variable and the graphical representation of the plastic deformations histogram, Figure 5, confirms the validation of the hypothesis of the distribution's normality.

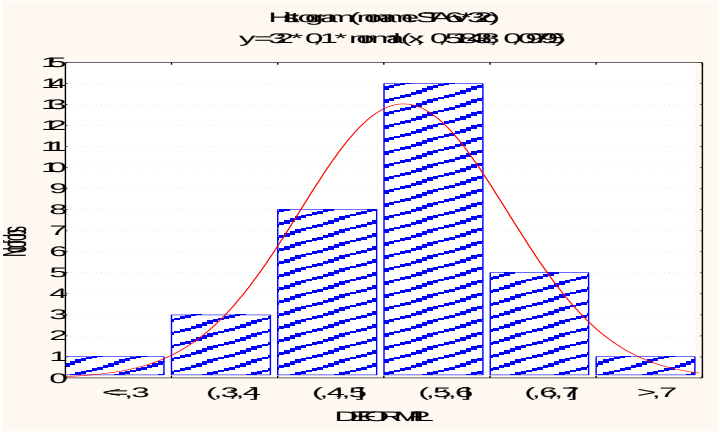


Figure 5: Plastic deformation histogram.

4. The correlation coefficients and the graphical representation of the regression equations.

The problem of choosing a single value that features the information given by a number of observations or independent measurements over a variable was solved by using the average value or the median.

The problem of determining the straight-line equation coefficients that better adjust three or more non-collinear points from a xoy plan, whose coordinates are the pairs of values corresponding to two variables X and Y, was generalized in two ways:

- for three variables it is searched the best three variable equation (the best plan in the three-dimensional space) and for more than three variables it is searched the best in more than three variable equation (the best hyper plan in a hyperspace);
- the equations from the above-mentioned situation can be linear or not, in which situation it is searched for the best curve in plan, the best surface from the three-dimensional space or the best hyper- surface from hyperspace;

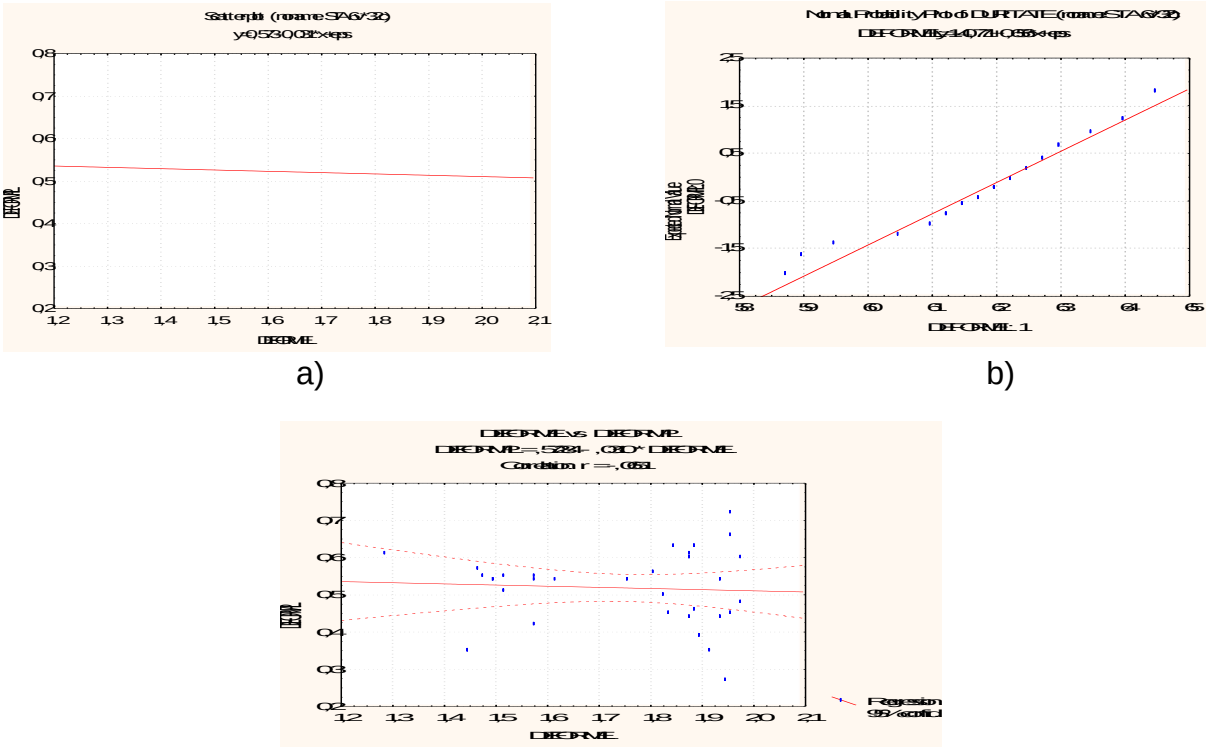
The problems of determining the best measures of the alignment tendency and of the best equation of the linear or non-linear regression are connected to each other and together to the problem of choosing the best measurement of dispersion. The solutions of all these three problems depend on the error or on the residue distribution. These represent the deviations of the observed values from those predicted by the regression equation. If we suppose that the values of the independent variables are known exactly and that the values of the dependent variable are distributed normally, then the average is the best measure of the alignment tendency, the standard deviation is the best measure of dispersion and the method of the smallest squares is the best method to adjust the regression equation, [4].

Once found the value of the correlation coefficient and the regression equation it can be defined a trust interval for these.

By using the STATISTICA, [5] specialized soft we can rapidly obtain a multitude of these kinds of graphical representations.

For example, the regression between the plastic and elastic deformation for all the 32 analyzed tests is presented in Figure 6.a, and the regression between the same variables but only for distinct hardness is represented in Figure 6.b. Building a trust

interval of the same regression, and also the correlation coefficient is presented in Figure 6.c.

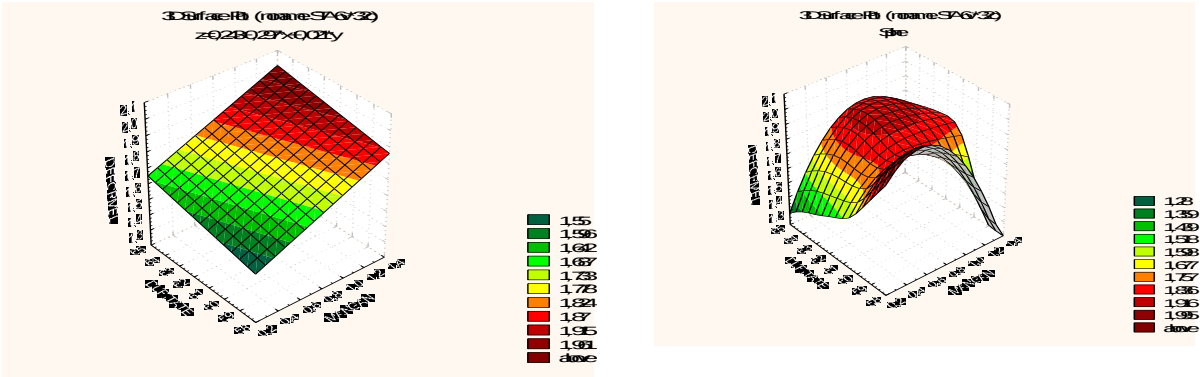


a) b)

c)

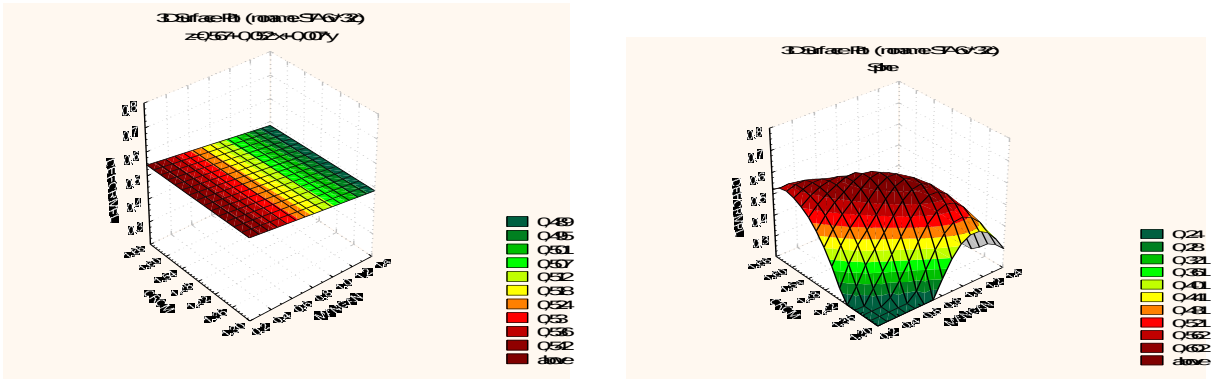
Figure 6: The regression straight-lines for the plastic and elastic deformations
a-for all the tests; b-only for different hardness; c-with trust interval.

Considering the three-dimensional space, we can determine the regressions between the elastic deformation, Figure 7, respectively the plastic one, Figure 8 and any other two of the independent variables: hardness, Carbon, Manganese or Chrome content.



a) b)

Figure 7: The regression surface for the elastic deformation according to the hardness and the Manganese percentage
a-linear, b-spline.



a) b)

Figure 8: The regression surface for the plastic deformation according to the Chrome and the Manganese percentage
a-linear, b-spline.

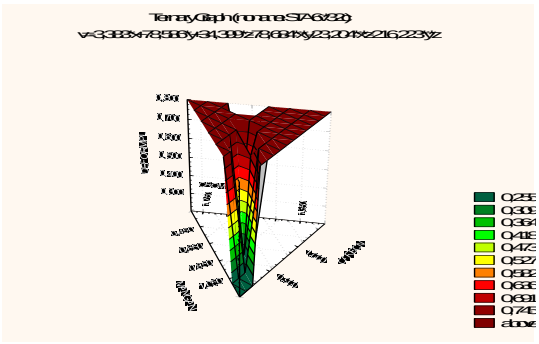


Figure 9: The regression hyper surface between the plastic deformation and the Carbon, Manganese and Chrome content (ternary graph).

In hyperspace, it can be determined the regression hyper-surface between the plastic deformation and the Carbon, Manganese and Chrome content by a ternary graph as the one from Figure 9.

5.**Conclusions**

One of the purposes of the scientific research is to create a model that could allow the passage from the specific conditions of the experiment to the result that should be obtained in different conditions without being necessary new experiments under new conditions.

For example, in our case, it is interesting to know what would be the elastic and plastic deformations of some samples identically to those from our experiment, but where the hardness and the chemical composition are different.

The mathematical statistics could be really useful in these types of analysis, if we have a sufficient number of data about the variables which intervene in the considered experiment.

It is also necessary the verification of the statistical hypothesis. The theory of probabilities and the mathematical statistics cannot offer the exact ideal model but they can offer the conclusions with the biggest probability or chance of realization.

The analysis made inside these experiments is valid in what regards the random character of the deformations and of the hardness, but some reserves are imposed on the chemical composition, because of the small number of data obtained in this direction. The paper can be considered a beginning, the analysis can be repeated by considering the alloying elements as distinct situations, instead of being random variables, Figure 10.

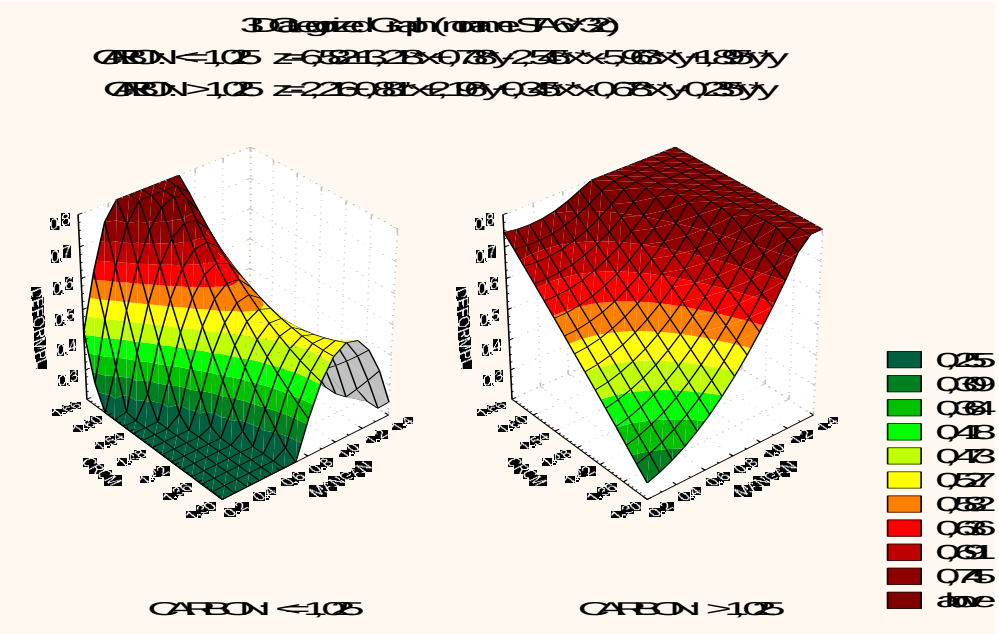


Figure 10: The regression surface for the plastic deformation according to the Manganese and Chrome percentage for the two percentage of Carbon.

The "STATISTICA" specialized program, used for realizing this paper, offers an impressing number of opportunities in what regards the approach by modern methods of the probability theory and of the mathematical statistics of the technical phenomena.

Bibliography

[1] NESTOR, T. IACOBESCU, F. PATRAS-CICEU, S., ALACI, S., PRODAN. D, - The Methods for the Determination of the Static Hysteresis at Contact Strain, VAREHDVIII, Suceava, June 1996
[2] NESTOR T., PRODAN D., PATRAS-CICEU S., ALACI S., PINTILIE D., - Stand for the Determination of the Static Hysteresis at Contact Strain, VAREHDVIII, Suceava, June 1996
[3] PANEL, L., TOKURA, S., HAGIWARA, I., - A Study for the influence of Work Hardening on Bending Stiffness of Truss, Transactions of the ASME, Journal of Applied Mechanics, January 2010
[4] MIHOC Gh., CRAIU V., - Treaty of Mathematical Statistics, Correlation and Linear Regression, Vol. IV, Academiei R.S.R. Publishing House, Bucharest, 1981.
[5] QUIK STATISTICA, Statsoft, http://www.statsoft.com